WORKBOOK
THE DYNAMICS OF MECHANICAL SYSTEMS

LUBLIN 2014

Projekt współfinansowany ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego

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1. DETERMINING THE CENTER OF GRAVITY USING A SECOND-CLASS LEVER METHOD

1.1. Theoretical background

The concept of center of gravity is very often confused with the concept of center of mass. The center of gravity of an object, or a system of objects is a point, in which the total weight of an object is concentrated. Whereas an object's, or group of objects' center of mass is a point, in which the total mass is concentrated, in a system description as a point mass. If an object is located in a uniform gravitational field, the center of gravity is identical to the center of mass.

A second-class lever method is based on the state of equilibrium of a flat parallel force system. Conditions of equilibrium of such a system are as follows:

$$\sum_{i=1}^{n} F_{iy} = 0, \ \sum_{i=1}^{n} M_i = 0 \tag{1.1}$$

Because determining the reaction force in point A (Figure 1.1) is not the aim, a equation of force moments will suffice. Assuming that the weight of the studied object (a person) is Q, and the weight of the berth is G, the momentum of forces equilibrium equation in relation to point O can be expressed in the following equation:

$$\sum_{i=1}^{n} M_{iO} = Qr + Gl/2 - R_{QG}l = 0 \tag{1.2}$$

where R_{QG} stands for the response caused by the weight of the object and the berth. It ought to be noticed that the response caused by the berth itself R_G also fits the momentum equation:

$$\sum_{i=1}^{n} M_{iO} = Gl/2 - R_G l = 0 \tag{1.3}$$

According to the superposition principle, the response produced by a person and the berth is a sum of the responses produced by the person only R_Q, and by the berth R_G. Taking into account the above dependencies, the center of gravity of the objects equals:

$$r = R_Q l / Q \tag{1.4}$$

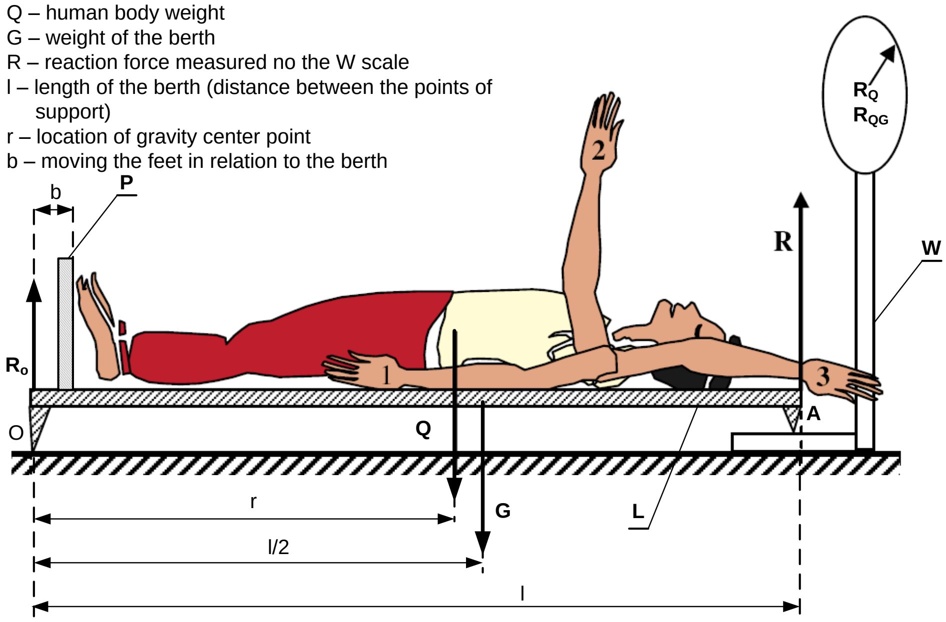

Figure 1.1. A second-class lever method

1.2. Exercise description

The exercise should be performed in a laboratory station equipped with a berth (L) and a scale (W). The berth's one end is on the floor in point O, the other end (point A) lays on the scale (W).

Exercise objective

The aim of the exercise is to calculate the location of the center of weigh of a given object (a person) using the second-class lever method.

Course of exercise

In a laboratory station shown in Figure1.1, the studied object should be placed in 3 different arm positions, as shown in the Figure 1.1, and each time the response R in the point of support (A) is measured.

Results analysis

The report should include the following information: the topic and objective of the exercise, the layout of the measuring work stand, the results chart, calculations and conclusions.

2. MODELLING OF OBLIQUE PROJECTION IN A RESISTING MEDIUM

2.1. Theoretical background

Let us consider a material point A with mass *m* initializing a motion in a uniform and constant gravitational field, with the initial velocity v_0, and angle ($^{\pi}/_{2} + \alpha$ to the field acceleration vector. Let's assume that the material point in motion, apart from gravity force, is also subjected to resistance force R, directly proportional to the square of its instantaneous velocity, and directed in the opposite direction to the velocity vector - see Figure 2.1.

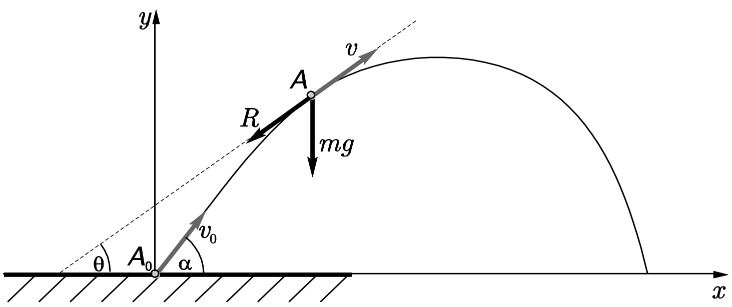

Figure 2. 1. Parabolic throwing in a gravitational field taking the resistance of the medium into account

The motion's equation of the given point in the vector form can be presented as follows:

(2.1)

Drawing vectors in tangent and normal direction to the motion path, and taking into account the definition of tangent and normal acceleration, we obtain:

— — (2.2)

where ρ stands for the radius of curvature in a given point of the path **( ),** *s* is the curvilinear coordinate measured along the path, is the motion's instantaneous velocity, and *k* is the air resistance factor.

It is not possible to find the closed-form solution of the point's motion path (i.e. the functional equation $y = f(x)$) from the above dependencies. It is possible however, to express this equation in the parametric form, with time *t* as the parameter, on the basis of the equation:

——— ——— (2.3)

The location coordinates can be expressed as integral dependence:

, (2.4)

where the function f(θ) appearing in the above dependencies can be stated as:

(2.5)

The integrals (2.3) and (2.4) cannot be expressed by elemental functions, that is why it is necessary to use the approximate numerical methods to calculate them.

2.2. Exercise description

The exercise should be performed on a computer, using MATLB environment and Simulink.

Exercise objective

The aim of the exercise is to establish the curvilinear motion path of a material point in the resisting medium, on the example of parabolic throwing. The ballistic curve will allow to determine the range and height of such throwing.

Course of exercise

Run MATLAB/ Simulink program and draw a numerical model simulating the discussed problem. Using the drawn model, determine the range and height of throw for 3-4 different angles for the initial velocity and two different initial velocities of the throw.

Results analysis

The report should include the following elements: the topic and aim of the exercise, the flow chart of numerical **model** drawn in MATLAB/Simulink environment, the ballistic curves found, the results chart of the range and height of the throw, and conclusions.

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3. ANALYZING THE OSCILLATION OF A MATHEMATICAL AND PHYSICAL PENDULUM IN LINEAR AND NON-LINEAR APPROACH

3.1. Theoretical background

A physical pendulum (Figure 3.1) is a rigid body, which can move around the horizontal axis going above its center of gravity. An important property of a pendulum is that its vibration period does not depend on the maximum displacement, that is, the amplitude. This dependency however, is only true for small angles of displacement of the pendulum, because then the motion equation (with a close enough approximation) can be expressed as a linear differential equation:

$$\ddot{\varphi} + \frac{mgl}{I_C}\varphi = 0 \tag{3.1}$$

The expression $\omega_o = mgl/I_c$ is the square of the normal mode oscillation frequency in linear approach, whereas I_c is the mass moment of inertia about the rotation axis through point C:

$$I_c = \frac{m}{2}\ R^2 + 3r^2 \tag{3.2}$$

Thus, the pendulum's vibration period is defined by the equation:

$$T_l = 2\pi \Big/ \sqrt{\frac{mgl}{I_C}} \tag{3.3}$$

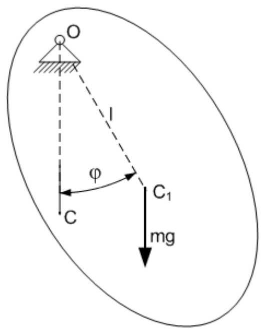
If the pendulum's angle of displacement is not small enough (larger than 5^o), the linear equation (3.1) looks like this:

$$\ddot{\varphi} + \frac{mgl}{I_C}\sin\varphi = 0 \tag{3.4}$$

The equation (4) is non-linear due to the pendulum's angle of displacement φ, and its solving requires some approximation. In this case, the pendulum's vibration period in the non-linear approach looks like this:

$$T_n = \frac{2\pi}{\omega_o}\left(1 + \frac{1}{4}\sin^2\frac{\varphi_o}{2} + \frac{9}{64}\sin^4\frac{\varphi_o}{2} + \frac{25}{256}\sin^6\frac{\varphi_o}{2}\right) \tag{3.5}$$

a) b)



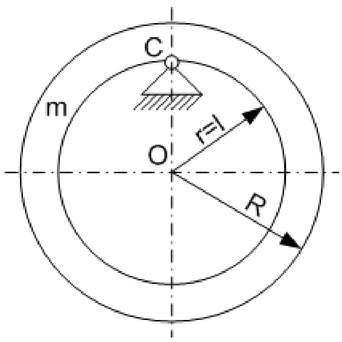


Figure 3.1. Physical pendulum (a), station layout (b)

3.2. Exercise description

The experiment should be performed using a ring (physical pendulum), as presented in the layout shown in Figure 3.1b. The calculations should be made using the available programs (Matlab, Excel).

Exercise objective

The aim of the exercise is an experimental measurement, and an analytical calculation of the pendulum's (ring's) normal mode vibration periods, using linear and non-linear equations.

Course of exercise

In the experiment station shown in Figure 3.1b, determine the vibration period, and then calculate the theoretical values based on the motion equation in the linear and non-linear form.

Results analysis

The report should include the following information: the topic and aim of the exercise, the layout of the measuring station, the results chart, calculations and conclusions.

4. FORCED VIBRATIONS OF SINGLE DEGREE OF FREEDOM SYSTEMS.

4.1. Theoretical background

Forced vibrations are vibrations caused by external forces exerted on a system. A system vibrates with the exciter frequency, which can have any value, independent of the system's natural frequency. Examples of systems of single degree of freedom (*1DOF - Single Degree of Freedom*), forced by a harmonic force $F(t)=A\cos(\omega t)$, are shown in Figure 4.1a and 1b, where A is the excitation amplitude, ω is the excitation frequency.

(a) *(b)*

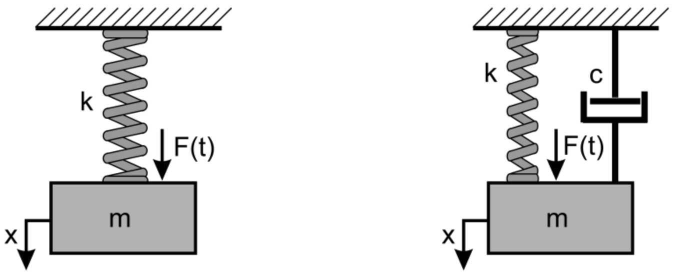


Figure 4.1. Non-damped system (a), and damped system (b) caused by harmonic force.

In a linear mechanical system, the normal mode vibration frequency (as well as the vibration period T) depends on the structural parameters of the system (its stiffness, mass); it does not depend on the initial conditions. For both cases shown in Figure 4.1, the normal mode vibration frequency and vibration period is

$$\omega_0 = \sqrt{k/m}, \quad T = 2\pi/\omega_0, \tag{4.1}$$

where k is the stiffness of the spring, and m stands for the mass of the hanged bob. The c parameter is the viscous damping.

If the normal mode vibration frequency equals the excitation frequency, a phenomenon of resonance occurs, which manifests itself with a sudden increase in vibration amplitude. Introducing even light damping causes the normal mode vibration amplitude to decrease abruptly, and only the vibrations of the excitation frequency remain.

4.2. Exercise description

The aim of the exercise is the analysis and simulation of a normal mode forced vibration in a single degree of freedom system, and to determine the characteristics of amplitude and frequency, and the frame curve. Additionally, an influence of damping on the vibration amplitude will be studied.

Course of exercise

The practical exercise comprises in creating a numerical model of a mechanical system with single degree of freedom (1DOF) forced by harmonic force *F(t)*, shown in Figure 4.1b. The equation of the system's motion is:

$$\ddot{x} + \alpha \dot{x} + \omega_0^2 x = q \cos \omega t \qquad (4.2)$$

where $\alpha = c/m$ (the damping to mass ratio), $q = A/m$ (the abruptly amplitude to mass ratio). Using Matlab (or Matlab-Simulink), create a numerical model based on the equation (4.2). Then, perform a numerical study for the system's parameters and the simulation conditions as given by the teacher. In order to automatize the procedure of drawing the resonance curves, you may loop calculations.

Results analysis

Perform the numerical simulation in order to draw the resonance curves (for different values of system damping α). Show the results on an amplitude-frequency chart. Determine the frame curve and assess the influence of damping on the amplitude. Describe the results and conclusions in a laboratory report.

5. DRAWING A MODEL OF NORMAL MODE VIBRATIONS IN A TWO DEGREES OF FREEDOM SYSTEM

5.1. Theoretical background

The normal mode vibrations play a significant role in the mechanics of a construction. They allow to identify the basic dynamic characteristics of a studied structure. The normal mode vibrations are caused by a temporary disturbance, after which the system continues to vibrate without the use of external force. Without damping, the normal mode vibrations in a system would not terminate and would be infinite. In reality, the normal mode vibrations are suppressed by damping. It is an example of damped normal mode vibrations.

A real object is usually presented as a model in a simplified way. To give an accurate description of a system's dynamics, it is sufficient to study a relatively simple model, in a form of interconnected oscillators comprised of concentrated masses and massless spring elements. Take for example a model of a two degrees of freedom system, shown in Figure 5.1.

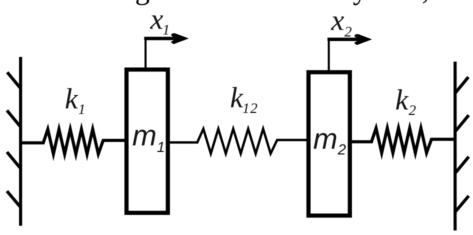


Figure 5.1. A model of two degrees of freedom system

The model is comprised of two oscillators with masses m_1, m_2 respectively, and stiffness k_1, k_2, connected by a spring with stiffness k_{12}.The motion of the masses is described with generalized coordinates x_1, x_2. The dynamics equations can be directly derived from Newton's second law of motion.

$$\begin{cases} m_1\ddot{x}_1 = -k_1 x_1 - k_{12}\left(x_1 - x_2\right) \\ m_2\ddot{x}_2 = k_{12}\left(x_1 - x_2\right) - k_2 x_2 \end{cases} \tag{5.1}$$

After moving all the expressions to the left-hand side of the equation (5.1), and then grouping with the coordinates $x_1$andx_2, we get

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{k}\mathbf{x} = \mathbf{0} \tag{5.2}$$

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where : $\mathbf{m} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $\mathbf{k} = \begin{bmatrix} k_1 + k_{12} & -k_{12} \\ -k_{12} & k_2 + k_{12} \end{bmatrix}$ are the square, symmetrical and positive definite matrices of inertia and stiffness, whereas $\mathbf{x} = x_1, x_2{}^T$, $\ddot{\mathbf{x}} = \ddot{x}_1, \ddot{x}_2{}^T$ are columned coordinate and generalized acceleration matrices (T index stands for transposed matrix). The basic parameters characterizing the normal mode vibrations are the frequency and mode shape, which can be determined analytically by solving the so-called eigenfunction.

5.2. Exercise description

The exercise should be performed on a computer station equipped with Matlab environment. For this purpose, in Matlab-Simulink system, build a numerical model using the selected Simulink libraries. Additionally, determine the vibrations frequency and mode shape on the basis of the equation (5.2)

Exercise objective

The aim of the exercise is to study the main characteristics of normal mode vibrations in two degrees of freedom systems and to learn the basics of Simulink.

Course of exercise

Using Matlab-Simulink library blocks, e.g. integrators, adders, oscilloscopes, etc., determine the time series of normal mode vibrations for different initial conditions. Solve the analytic eigenfunction, that is, determine the vibration frequency and mode shape for values given by the teacher. Determine the initial conditions in which "pure" first and second mode shape will be excited. Run a numeric simulation for such a case.

Results analysis

The report should include: , system model in Simulink, the vibration frequency and mode shape determined analytically for values given by the teacher, the vibrations' time series calculated from Simulink model confirming the adequacy of the analytical results, final conclusions.

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6. STUDYING THE DYNAMICS OF A CAR SUSPENSION MODEL. DYNAMIC VIBRATION ABSORBER

6.1. Theoretical background

In the analysis of multiple degree of freedom systems, such as a car suspension, simplified, linear mathematical models are used, which are approximations of the real systems. Assuming there is a symmetry in the workings of a car suspension system, the analysis can be narrowed to just one axle and one wheel supporting one-fourth of a car (Figure 6.1a). Using Newton's second law of motion, the differential equations of motion of one fourth of a car for unsprung m_u and sprung mass m_s are stated as:

$$m_s\ddot{y}_s + k_s \; y_s - y_u \; = 0$$
$$m_u\ddot{y}_u + k_t y_u - k_s \; y_s - y_u \; = k_t y_0 \qquad (6.1)$$

where the road profile excitation: yo=Asin(Ωt), Ω constitutes the circular frequency of the excitation and is stated as Ω=2πν/L.

a) b)

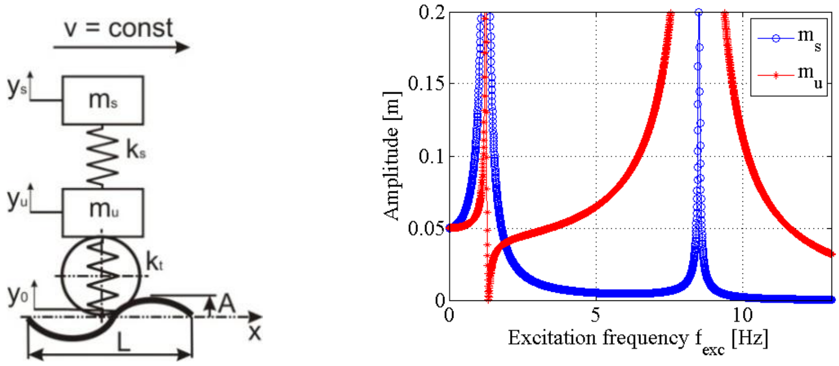

Figure 6.1. One-fourth of the car suspension model (a), and (b): amplitude - frequency characteristics for the given data: sprung mass m_s=560[kg], *unsprung mass* m_u=102[kg], *excitation amplitude* A=0.05[m], *suspension stiffness* k_s=40000[N/m] *and tire stiffness* k_t=250000[N/m].

For the above equations, search for independent solutions which would fit the vibrations according to the forcing function $y_0(t)$.

Using the determinant method, the dependency of unsprung m_u and sprung m_s mass vibration amplitude can be found. For the determined number of vibrating masses interconnected with springs (suspension), normal mode vibration frequency occurs, which can be calculated by solving the so-called characteristic equation. The results of the analytical study has been shown as graphs (Figure 6.1b).

Amplitude-frequency characteristics present three characteristic points: the first and second resonance and anti-resonance. The effect of anti-resonance almost completely damps the vibrations of the unsprung mass m_u, which is affected by an exterior excitation. The excitement energy y_0 is transferred to the sprung mass m_s.

6.2. Exercise description

The exercise should be performed by creating a numerical model of a car suspension system using Simulink module in Matlab environment. Following teacher's instructions, run a series of simulations for different work conditions of the model with respect to the characteristic values of exciting frequencies as shown in Figure 6.1b.

Exercise aim

The aim is to verify the results of analysis using numerical simulations, and to observe the effect of vibration damping and how the model works with resonance.

Exercise procedures

Using the program for simulations, enter the exciting function parameters, that is, amplitude of excitation *A[m]*, wave length *L[m]*, car's velocity *v[km/h]*, sprung mass m_s*[kg]* and unsprung mass m_u*[kg]*, suspension stiffness k_s*[N/m]*, and tire stiffness k_t*[N/m]*.Run the simulation for a few different velocities *v* showing how the system works in first and second resonance and anti-resonance. Note the results in the measure report.

Results analysis

The report should include the following information:

1. Exercise topic;
2. Exercise aim;
3. A layout of the numerical models of each block;
4. The measurements table for parameters given by the teacher
5. Attached report of the measurements.

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7. DRAWING A MODEL OF DAMPED MECHANICAL SYSTEMS

7.1. Theoretical background

When designing a mechanical construction, the system's components and its surrounding, e.g. the floor, should have an adequate vibration isolation. For such a purpose, spring elements (such as: coil springs, rubber elements, pneumatic springs), or damping elements (e.g. friction dampers, hydraulic dampers) are commonly used. Different methods of energy dissipation can be stated in the form of the following damping force models F_t:

- friction dampers - damping with dry friction (Coulomb's model), where the force $F_t = f_0 sgn(v_w)$ depends on the direction of the relative speed v_w of absorber's elements,. In numerical studies, function sgn is often replaced by a continuous function, e.g. tanh. Thus, damping force can be stated as $F_t = f_0 \tanh(\lambda v_w)$, where λ coefficient determines the shape of the curve in the area where the value changes from $-f_o$ to $+f_o$.
- hydraulic dampers - viscous damping model. Damping force is proportional to the relative speed v_w, $F_t = cv_w$. The damping coefficients can have a constant value (linear characteristics), or, as in a car suspension, different values when the absorber is pressed c_1,and when it is stretched c_2 (partly linear characteristics).
- the modern damping methods, e.g. magnetorheological dampers (MR) - Bingham's model, in which damping force stems from the parallel connection between the friction shock absorber and viscous shock absorber, $F_t = f_0 sgn(v_w) + cv_w$ $F_t = f_0 sgn(v_w) + cv_w$ This is the simplest model of an MR damper. In literature, there is a number of its modifications which modify the influence of additional factors. For example the hysteresis effect is accounted for in Bouc-Wen's model, which can be described by equations: $F_t = cv_w + \alpha z$, where $\dot{z} = -\gamma |v_w| z |z|^{n-1} - \beta v_w |z|^n + A v_w$.

7.2. Exercise description

Exercise objective

Creating a flow chart of a single degree of freedom system model in Matlab-Simulink. Compiling amplitude-frequency characteristics for different shock absorbing elements being used.

Course of exercise

Figure 7.1 shows a layout of the studied one degree of freedom system. When performing the exercise, complete the following steps*:*

- derive differential equation describing the system with the use of different absorbers, e.g. partly linear model, Bouc-Wen's model,
- on the basis of the derived differential equation, create a flow chart of the model in Matlab-Simulink,
- run numerical simulations necessary to comply the characteristics.

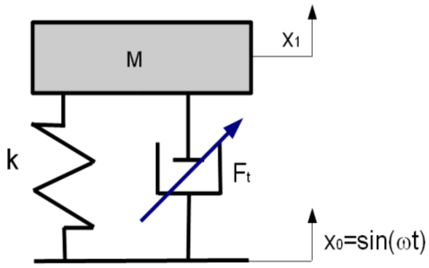

Figure 7.1. A model of a single degree of freedom system

Results analysis

The exercise report should include: introducing information (who performed the exercise, exercise number, exercise aim), a layout of the studied system, differential equations derivations, Matlab-Simulink flow charts, scripts structure (m-files) used for numerical simulations, amplitude-frequency characteristics, conclusions.