WORKBOOK

MECHANICS OF MATERIALS AND ELEMENTS OF ENGINEERING STRUCTURES

LUBLIN 2014

Projekt współfinansowany ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego

Authors: Sylwester Samborski, Andrzej Teter and Marcin Bocheński

Desktop publishing: Sylwester Samborski, Andrzej Teter and Marcin Bocheński
Technical editor: Sylwester Samborski
Figures: Sylwester Samborski, Andrzej Teter and Marcin Bocheński
Cover and graphic design:

Publikacja współfinansowana ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego w ramach projektu
Inżynier z gwarancją jakości – dostosowanie oferty Politechniki Lubelskiej do wymagań europejskiego rynku pracy

Lublin 2014

First edition

TABLE OF CONTENTS

LIST OF MAJOR SIGNS AND ABBREVIATIONS

σ	– normal stress
τ	– shear stress
E	- Young's modulus
ν	- Poisson coefficient
G	- shearing modulus
φ	- angle of twist
I_z	- second moment of area

1. AXIAL TENSION, HOOKE'S LAW AND THE SUPERPOSITION PRINCIPLE

1.1. Theoretical background

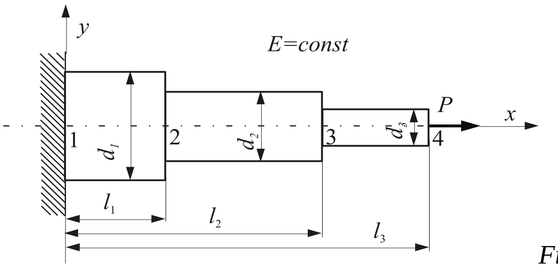

Figure 1.1. Illustration of a stepped bar in tension; thedimensions are marked.

In order to assess materials' mechanical properties, a static tension test is conducted. A normalized sample of a cylindrical shape is loaded with the axial force P. The sample's initial length is l_o and the cross sectional area is F_o. The resultant internal force is generated within the sample: $N=P$. It is a system of two forces that are in equilibrium. The measure of the internal forces' intensity is the normal stress equal to: $\sigma=N/F_{o.}$ .With the increase of the load P, the sample's actual length l is being measured and its elongation is being calculated: $\Delta l=l-l_o$ or $\varepsilon=\Delta l/l_o$. On the basis of such data, a tension graph is constructed. The initial part of the tension graph is a straight line described by Hooke's law:

$$\Delta l = \int_l \frac{N \cdot dx}{F\,E} = \int_l \frac{\sigma \cdot dx}{E} \qquad (1)$$

The material constant describing mechanical properties of the bar, called the Young's modulus, is marked by E. The value of the Young's modulus for steel equals 205 GPa. The threshold for applying Hooke's law is the proportionality limit R_H. The stresses accompanied by rapid deformations at a practically constant load value are called the yield strength R_e. The maximum stress value marked at the tension curve is called the tensile limit R_m. For strength calculations the superposition principle is applied. This principle states that the resultant effect of action of a complex system of loads is equal to the algebraic sum of direct effects of the loads acting separately. Furthermore, the following assumptions must be met: the system suffers small deformation, the material complies with Hooke's law and there is no coupling between the loads. The superposition principle may be applied for determining progressive elongation of the bar, illustrated in the Fig. 1.1. The total elongation of the bar equals the sum of

Projekt współfinansowany ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego

elongations of its every steps. The value of the internal force along the entire length of the bar is constant: $N=P$. For the adopted dimensions (Fig. 1.1), the following formula is obtained:

$$\Delta l = \frac{N \cdot l_1}{EF_1} + \frac{N \cdot (l_1 - l_2)}{EF_2} + \frac{N \cdot (l_2 - l_3)}{EF_3} \qquad (2)$$

1.2. Description of the exercise

1.2.1. Objectives of the exercise

The objective of the exercise is to describe the effects occurring during statically determinate loading of a stepped bar working under axial tension or compression, as well as to compare, for the given case, the elongation values under an increasing load, obtained through experimental research and through calculations based on the superposition method.

1.2.2. Outline of the exercise

Draw a sketch of the analysed bar and dimension it. Then fix the specimen in the clamps of the testing machine and measure the total elongation of the bar under the given forces. Finally, determine the theoretical elongation by using the superposition method.

1.2.3. Preparation of the report

Draw a dimensioned specimen. For the given force P, draw a stress diagram. Compare the results of the experimental and the theoretical research. Assess the error. Include the following in the report: the subject, objectives of the exercise, a diagram of the test stand, a summary of the results, calculations and conclusions.

Projekt współfinansowany ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego

2. SHEARING OF PERMANENT JOINTS

2.1. Theoretical background

The permanent joint is a joint between structural components that cannot be undone without destroying the connecting element (e.g. a rivet or a weld).

The types of permanent joints are as follows:

- riveted (Fig. 2.1.),
- welded,
- adhesive,
- other.

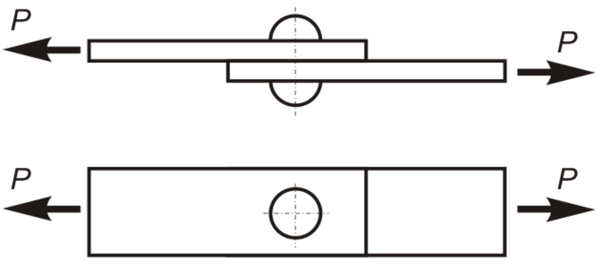

Figure 2.1. Riveted joint as an example of the permanent joint

In strength calculations for permanent joints, the following strength conditions are usually checked:

$$\sigma_{\mathrm{r}} = \frac{P}{A_{\mathrm{r}}} \le k_{\mathrm{r}} \quad (1)$$

$$\tau_{1} = \frac{P}{A_{\mathrm{t1}}} \le k_{\mathrm{t}} \quad (2)$$

$$\tau_{2} = \frac{P}{A_{\mathrm{t2}}} \le k_{\mathrm{t}} \quad (3)$$

$$\sigma_{\mathrm{d}} = \frac{P}{A_{\mathrm{d}}} \le k_{\mathrm{d}} \quad (4)$$

where: σ and τ are normal and shear stresses, respectively; P – the joint loading force, A – the cross section area and k is the allowable stress.

2.2. Description of the exercise

2.2.1. Objectives of the exercise

The objective of the exercise is to empirically measure the strength of a permanent joint by means of quasi-static tension, after having calculated its theoretical strength by checking the conditions from (1) to (4), depending on the type of the joint.

2.2.2. Outline of the exercise

Draw the permanent joint given in the exercise and dimension it. On the basis of the strength conditions, calculate the maximum load. Then carry out a tensile test until its breakdown. Carry out a visual examination of the broken specimen.

2.2.3. Preparation of the report

The report on the test conducted should include strength calculations of the tested joint, a joint loading force diagram, and sketches of the dimensioned specimen, before and after the breakdown, as well as results' analysis and conclusions.

3. DETERMINATION OF SECOND MOMENTS OF AREA OF BEAMS' CROSS-SECTIONS

3.1. Theoretical background

The value of the second moment of area of a cross-section can be derived directly from the definition: **the second moment of area** of a figure with respect to any given axis is the integral over the area calculated from the product of the square of the element's distance *r* from the axis and the elementary area *dF*:

$$I = \int_F r^2 dF \qquad (1)$$

For the adopted coordinate system *xy* (Fig. 3.1) the second moment of area of the figure with respect to the *z* axis can be represented by the relation:

$$I_z = \int_F y^2 dF \qquad (2)$$

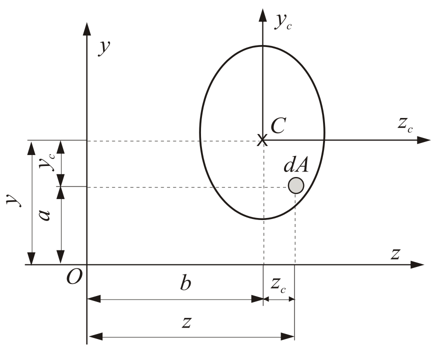

Figure 3.1. Derivation of the second moment of area

The Steiner's theorem: the second moment of area of a plane figure with respect to the axis placed in the distance *a* from the figure's centre of gravity is equal to the second moment of area with respect to the parallel axis passing through the centre of gravity, increased by the product of the total area of the figure and the square of the distance between the two axes:

$$I_z = I_{zc} + a^2 A \qquad (3)$$

The second moment of area of a complex figure equals the sum of the moments of inertia of the component figures with respect to a given axis.

In order to determine the approximate value of a mechanical system's stiffness, the Geiger's formula, known from mechanics, can be applied. The whole measuring procedure consists in dividing the analysed beam of length *L* into *n* number of equal sections. Then a displacement sensor is placed in the centre of gravity of each interval, a constant load is applied and the deflection is measured.

$$I_z = \frac{n\,PL^3}{\pi^4 E \sum_{i=1}^{n} y_i(P)} \tag{4}$$

If the beam is composed of one part, the assessment error of the second moment of area will be considerable. On the other hand, division into three parts will be accurate enough.

3.2. Description of the exercise

Experimental tests are carried out at a test stand including a simply supported steel beam of the length *L*. In the selected point a student places the displacement sensor, fixed on a magnetic tripod,. The load hanger should be placed in the direct vicinity of the displacement sensor and its indication should be read.

3.2.1. Objectives of the exercise

The objective of the exercise is to determine the approximate value of the second moment of area of the bent beam's cross-section by application of the Geiger's method.

3.2.2. Outline of the exercise

Draw the cross-section of the beam and dimension it. Mark the centre of the beam on its surface as (1). Place the displacement sensor in the point (1) and reset it. Place the tray with the load *P* in the direct vicinity of the point (1) and read the displacement value from the sensor. Following the tutor's guidelines, divide the beam into few parts and repeat the measurement procedure.

3.2.3. Preparation of the report

Taking into account the definition of the second moment of area (1) calculate its for a given beam's geometry. Then, basing on the experimental tests, assess the subsequent approximate values (4). Write down the results of the measurements and tests in a table. Assess the error. Include the following in the report: the subject, objectives of the exercise, a diagram of the test stand, a summary of the results, calculations and conclusions.

KAPITAŁ LUDZKI NARODOWA STRATEGIA SPÓJNOŚCI
UNIA EUROPEJSKA EUROPEJSKI FUNDUSZ SPOŁECZNY
Projekt współfinansowany ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego

4. TORSION. DETERMINATION OF THE ANGLE OF TWIST

4.1. Theoretical background

Torsion is a simple case of loading, where the external forces are reduced to a couple of forces, whose operating plane is tangent to the cross-section. The moment of the couple, called the twisting moment, is parallel to the axis of a shaft. Let us consider a shaft of diameter *d* and length *l*, subjected to a twisting moment M_s (Fig. 4.1).

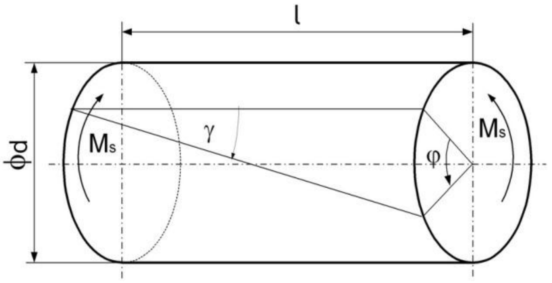

Figure 4.1. Deformation of a shaft under torsion

Due to the loading, the shaft's generatrix experiences deformation, becoming a helix with the angle of inclination γ, whose value is constant along the entire shaft. Neither the length nor the diameter change during the process, and all the sections perpendicular to the axis of the unloaded shaft remain plane also after loading. The angle φ by which the shafts's end cross-sections will rotate in relation to each other is represented by the formula:

$$\varphi = \frac{M_s l}{GJ_0} \tag{1}$$

where: *G* – shear modulus (Kirchhoff's modulus), J_0 – the polar moment of area of the cross-section.

For the circular section, this moment equals:

$$J_0 = \frac{\pi r^4}{2} = \frac{\pi d^4}{32} \tag{2}$$

When dealing with a hollow shaft, which is a pipe with the external radius *R* and the internal radius *r*, the formula (2) takes the form:

$$J_0 = \frac{\pi\left(R^4 - r^4\right)}{2} = \frac{\pi\left(D^4 - d^4\right)}{32} \qquad (3)$$

The angle γ can be calculated with the following formula:

$$\gamma = \varphi \frac{r}{l} = \frac{M_s r}{G J_0} = \frac{\tau_s}{G} \qquad (4)$$

The maximum torsional (shear) stresses used in this formula are described by the relation:

$$\tau_s = \frac{M_s}{W_0} \qquad (5)$$

where: W_0 – the polar section modulus of a cross-section equal to the quotient of the polar moment of area of the cross-section and the external radius (R) of the shaft.

4.2. Description of the exercise

4.2.1. Objectives of the exercise

The objective of the exercise is to determine the angle of twist of a pipe, depending on the value of the twisting moment applied by means of a couple of forces.

4.2.2. Outline of the exercise

Draw and dimension a diagram of the tested pipe, along with the loading system. Reset the angle of twist measuring system. Then, following the tutor's guidelines, load the system and read the value of the angle of twist.

4.2.3. Preparation of the report

Determine the values of the angle of twist for each load. Illustrate theoretical and experimental data in a graph: $\varphi=\varphi(M_s)$. Assess the error. Include the following in the report: the subject, objectives of the exercise, a diagram of the test stand, a summary of the results, calculations and conclusions.

5. DETERMINATION OF MATERIAL CONSTANTS FOR A BENT ISOTROPIC BEAM

5.1. Theoretical background

A beam is a bar subject to bending, involving or not shearing forces. In the general case of bending, one can discern simple (flat) bending, which takes place when the vector of the bending moment M_g coincides with one of the principal axes of a cross–section (Fig. 5.1). Otherwise diagonal bending takes place. Pure bending is a special case of bending when shearing forces T do not occur.

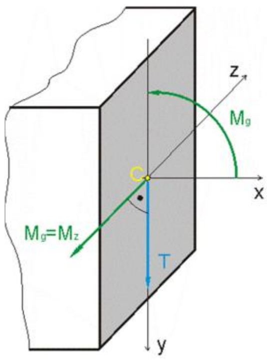

Figure 5.1. Internal forces in simple bending

Deflection of a beam subjected to three-point bending (Fig. 5.2) is represented by the formula:

$$f = \frac{Pl^3}{48EI_z} \qquad (1)$$

where: P stands for the loading force, l– the distance between the supports, I_z – the second moment of area of the beam's cross-section with respect to the axis z. Having modified this relation, one can empirically determine the values of the tangent Young's modulus in bending E_b for the tested beam; it is enough to measure the deflection f corresponding to the force P and to apply the relation (Cf. ASTM D790):

$$E = \frac{Pl^3}{48 f \, I_z} \qquad (2)$$

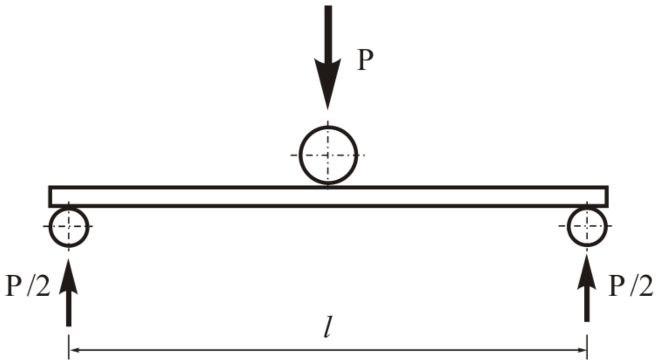

Figure 5.2. Three-point bending of a beam

5.2. Description of the exercise

5.2.1. Objectives of the exercise

The objective of the exercise is to experimentally determine Young's modulus by submitting a beam to three-point bending.

5.2.2. Outline of the exercise

Draw the tested beam and provide its dimensions. While loading the beam placed in a three-point bending device, read the maximum deflection values (in the middle) corresponding to the applied force. Compute the Young's modulus in bending.

5.2.3. Preparation of the report

The report should include the calculations for one load value, a sketch of the tested structure as well as a calculation scheme. Collect the results of theoretical calculations and tests in a table. Assess the error.

6. DETERMINATION OF BEAMS' DEFLECTION LINES

6.1. Theoretical background

In order to calculate the deflection at any given point of a beam, it is necessary to solve the differential equation of the deflection line:

$$\frac{d^2y}{dx^2} = \frac{M_g}{EI_z} \tag{1}$$

Solving the equation consists in integrating it twice and finding the constants of integration from the boundary conditions. In a general case, the differential equation of deflection line should be solved in every interval of internal forces' variation. The calculations are simplified by employing the Clebsch's procedure, which automatically meets the requirements for the structures continuity.

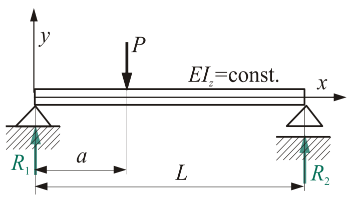

Figure 6.1. A diagram of the discussed multi-span beam

In case of a simply supported beam of length *L* and loaded with the vertical force *P* applied in the distance *a* from the support, the differential equation of the deflection line takes the form:

$$EI_z \frac{d^2y}{dx^2} = P \cdot \frac{L-a}{L} \cdot x \Bigg|_{x\in<0;a)} - P \cdot (x-a) \Bigg|_{x\in(a;l>} \tag{2}$$

The solution, obtained after double integration and determination of the constants of integration from the conditions: 1) for x=0, y=0 and 2) for x=L, y=0, can be written as:

$$EI_z y = -P \cdot \frac{(L-a)\cdot(2L-a)\cdot a}{6 \cdot L} \cdot x + P \cdot \frac{L-a}{L} \cdot \frac{x^3}{6} \Bigg|_{x\in<0;a)} - P \cdot \frac{(x-a)^3}{6} \Bigg|_{x\in(a;l>} \tag{3}$$

The maximum deflection of a bent structure is determined by finding the local extreme of the function describing the deflection line. Furthermore, it is necessary to examine deflections at the borders of the intervals.

In experimental measurements, the Maxwell's reciprocal theorem can be applied: „*For a linear elastic structure subjected to two forces of the same value, the displacement corresponding to the first force induced by the second force is equal to the displacement corresponding to the second force, caused by the first force.*". Application of this theorem allows to estimate the deflection line with the use of a single displacement sensor. It is enough to place the sensor at the point of load application and reset it. Then apply still the same load value in subsequent points. The deflection values read, in accordance to the Maxwell's reciprocal theorem, are equal to the deflection of a beam in the point of load application.

6.2. Description of the exercise

Experimental tests are conducted at a test stand consisting of steel beam having constant bending stiffness. The beam is simply supported at the ends and loaded in a chosen point with a weight placed on a hanger. The hanger, based on a roll, can move along the beam. The displacement sensor, fixed on a magnetic tripod, is placed at another point.

6.2.1. Objectives of the exercise

The objective of the exercise is to empirically determine the deflection line of the simply supported beam loaded with a vertical concentrated force.

6.2.2. Outline of the exercise

Measure the geometry of the beam. Mark measuring points on the surface of the beam. Place displacement sensors in the subsequent points and reset them. Put the weight on the hanger. Read the deflection value.

6.2.3. Preparation of the report

For the adopted coordinate system determine coordinates for the measuring points. Calculate theoretical values of displacement. Write down the results of calculations and measurements in a table. Assess the error. Include the following in the report: the subject, objectives of the exercise, a diagram of the test stand, a summary of the results, calculations and conclusions.

KAPITAŁ LUDZKI
NARODOWA STRATEGIA SPÓJNOŚCI

UNIA EUROPEJSKA
EUROPEJSKI
FUNDUSZ SPOŁECZNY

Projekt współfinansowany ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego

7. ANALYSIS OF THE STRESS STATE. MOHR'S CIRCLE

7.1. Theoretical background

Let us look at an element subjected to a plane state of stress (Fig. 7.1). The principal stresses σ_1 and σ_2 act along the principal directions of the element. These are the extreme normal stresses, which are not accompanied by shear stresses. Let us take a cross-section of the discussed element at an angle α and introduce normal stress σ_α as well as shear stress τ_α, which give static equilibrium of the internal forces:

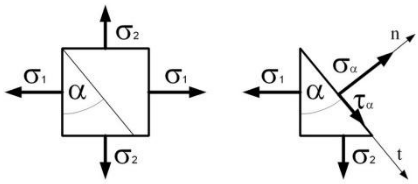

Figure 7.1. Two-dimensional stress state of the element

Then, derived from the conditions of equilibrium, one can write the relations:

$$\sigma_\alpha = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\alpha$$

$$\tau_\alpha = \frac{\sigma_1 - \sigma_2}{2}\sin 2\alpha \qquad (1)$$

The stresses σ_α and τ_α for any given angle α create a circle with the centre in the point that is the arithmetic mean of the principal stresses and with a radius equal to one half of their difference (Fig. 7.2).This circle is called the Mohr's circle for a two-dimensional state of stress. Additionally, for a structure where the values of normal and shear stresses are known, by means of the circle, it is possible to determine the direction and values of the principal stresses:

$$\sigma_{\frac{1}{2}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$tg 2\alpha = \frac{2\tau}{\sigma_x - \sigma_y} \qquad (2)$$

Projekt współfinansowany ze środków Unii Europejskiej w ramach Europejskiego Funduszu Społecznego

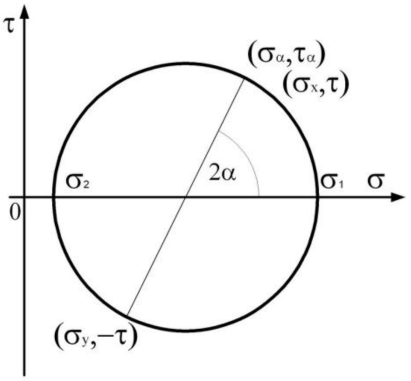

Figure 7.2. Mohr's circle for a plane state of stress

7.2. Description of the exercise

7.2.1. Objectives of the exercise

The objective of the exercise is to analyse the plane state of stress and determine normal and shear stresses in any given direction by application of the Mohr's circle.

7.2.2. Outline of the exercise

Draw the analysed structure along with the arrangement of strain gauges. Apply the loading force given by the tutor and read the strains recorded by the strain gauges.

7.2.3. Preparation of the report

Using the relation:

$$\sigma_1 = \frac{E}{1-\nu^2}\left(\varepsilon_1 + \nu\varepsilon_2\right),\ \sigma_2 = \frac{E}{1-\nu^2}\left(\varepsilon_2 + \nu\varepsilon_1\right) \qquad (3)$$

draw the Mohr's circle and then determine graphically and analytically values of normal and shear stresses for the cross-section oriented at the angle given by the tutor. Include the following in the report: the subject, objectives of the exercise, a diagram of the test stand, a summary of the results, calculations and conclusions.